HEREDITY IN HEAD FORM

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The recent discussion of Mendel's law has called renewed attention to the phenomena of heredity. In anthropology this problem has been discussed principally by Galton and Pearson who have treated their materials according to the method of correlations. The principle of this method requires two assumptions: first, that each parent has a certain influence upon the form of each offspring and that the amount of this influence remains the same in all cases, except that it is subject to chance variations; second, that the variability of the offspring does not depend on the form of the parent.[1] If Mendel's law holds good even in a restricted way in the human species, these assumptions would not be admissible on account of the dominant influence of one parent or the other, and the results of the method of correlation could be considered as rough approximations only.

If we assume that the influence of one parent dominates, we must expect to find a massing of the measurements of descendants at points corresponding to those of the parental measurements. Therefore, the greater the difference between the parents, the greater must be the variability of the offspring. The character of the laws of heredity can therefore be determined by an investigation of the question whether the variability of the offspring depends upon the difference between the parents.

I was led to take up this question by an observation which I had published in 1894, in a study of the half-bloods of the American and European races.[2] I found that in this case there is a decided tendency to develop a width of face that corresponds either to that of the European or to that of the American race, while intermediate forms are rare. This phenomenon conforms to Mendel's law.

[1] See Franz Boas, "The Cephalic Index," *American Anthropologist*, N. S., vol. I, pp. 448–461.

[2] *Popular Science Monthly*, October, 1894, pp. 761–770.

I discussed this question at length with Dr Maurice Fishberg of New York, whose studies of the anthropology of the Jews are so well known, and he had the great kindness to collect for the purpose of this investigation measurements of whole families of East European Jews. Anyone who has tried to collect similar data will appreciate the difficulties of such an undertaking and the labor involved. Dr Fishberg succeeded in collecting measurements of forty-eight families. Although this number is hardly sufficient to elaborate the problem in detail, a few interesting conclusions can be drawn.

The cephalic indices of the whole series are distributed as follows:

	Males						Females					Total	
	Fathers	Sons					Mothers	Daughters					
Age.		≧20	15–19	10–14	5–9	1–4		≧15	10–14	5–9	1–4	Men.	Women and Children.
73[1]	1	—	—	—	—	—	—	—	—	1	—	1	1
74	2	—	—	—	—	—	—	—	—	—	—	2	—
75	—	—	1	—	1	—	1	—	—	—	—	—	3
76	1	—	—	3	2	—	1	—	—	—	—	1	6
77	4	1	—	1	—	—	4	1	—	—	1	5	7
78	5	—	1	1	1	—	1	1	1	—	—	5	6
79	2	1	—	5	—	—	—	1	2	—	2	3	10
80	5	4	2	2	4	1	8	2	3	5	3	9	30
81	8	1	2	3	2	2	7	1	1	3	—	9	21
82	10	—	1	4	1	1	3	1	1	4	2	10	18
83	4	2	—	2	6	3	8	1	2	1	1	6	24
84	3	1	2	1	3	—	2	—	3	5	—	4	16
85	—	—	1	—	2	—	5	—	4	2	3	—	17
86	2	—	2	2	6	3	3	2	2	2	—	2	22
87	1	—	1	1	1	—	1	1	—	—	1	1	6
88	1	—	—	3	—	—	2	—	—	—	—	1	5
89	—	—	—	—	1	—	1	—	—	—	1	—	3
90	—	—	—	—	—	—	1	—	—	—	—	—	1
91	—	—	—	—	—	—	1	—	—	—	—	—	1
Cases	49	10	13	28	30	10	49	11	19	23	14	59	197
Aver.	81.2	81.2	82.7	82.0	83.2	83.6	82.9	82.2	83.1	82.6	82.9	81.2	82.7

This table shows that, while the heads of the men are a little more elongated than those of the women and children, the whole series is sufficiently uniform to admit of a comparison between the indices of individuals of different ages.

[1] The indices contain the values from 73.0–73.9, 74.0–74.9, etc.

If the material were sufficiently extensive, it would be easy to investigate directly the distribution of the indices of children whose parents have indices of certain fixed values. This is impossible with the limited material at our disposal. We can, however, study the relation between the variability of children and the differences between the parental couples.

Before entering into this question we will determine the distribution of variabilities in case Mendel's law should prevail.

According to a generalized form of Mendel's law we may expect some children to show the tendency to revert more strongly to the paternal than to the maternal type, while others exhibit the reverse tendency. This may be expressed in algebraical symbols, as follows: Let n be the total number of children, n_1 and n_2 respectively the number of children with dominant paternal and maternal type; r_{cf}, r_{cm} the coefficients of correlation of children with their fathers and their mothers among the series with dominant paternal type, r'_{cf}, r'_{cm} the corresponding coefficients among the series with dominant maternal type. Let x, y, z be the deviations of fathers, mothers, and children respectively; let [ ] indicate averages; and let finally σ_f, σ_m, σ_c be the standard variations of fathers, mothers, and children respectively.

Then we have for the series of n_1 children with dominant paternal type

$$[z] = r_{cf}\frac{\sigma_c}{\sigma_f}x + r_{cm}\frac{\sigma_c}{\sigma_m}y$$

$$[xz] = r_{cf}\sigma_c\sigma_f + r_{cm}\frac{\sigma_c}{\sigma_m}[xy].$$

In our case the correlation between fathers and mothers or the "assortive mating"

$$r_{fm} = \frac{[xy]}{\sigma_f\sigma_m} = +0.15.$$

This amount is so slight that it may be disregarded. Therefore —

In the same way

$$\left.\begin{array}{l}[xz] = r_{cf}\sigma_c\sigma_f \\ [yz] = r_{cm}\sigma_c\sigma_m\end{array}\right\} n_1 \text{ cases.}$$

For the series with dominant maternal type we find in the same way

$$\left.\begin{aligned}[z] &= r'_{cf}\frac{\sigma_c}{\sigma_f}x + r'_{cm}\frac{\sigma_c}{\sigma_m}y \\ [xz] &= r'_{cf}\sigma_c\sigma_f \\ [yz] &= r'_{cm}\sigma_c\sigma_m\end{aligned}\right\} n_2 \text{ cases;}$$

and for the total series

$$(1)\qquad [z] = \frac{\sigma_c}{n}\left\{(n_1 r_{cf} + n_2 r'_{cf})\frac{x}{\sigma_f} + (n_1 r_{cm} + n_2 r'_{cm})\frac{y}{\sigma_m}\right\}$$

$$(2)\qquad [xz] = \frac{n_1 r_{cf} + n_2 r'_{cf}}{n}\sigma_c\sigma_f$$

$$(3)\qquad [yz] = \frac{n_1 r_{cm} + n_2 r'_{cm}}{n}\sigma_c\sigma_m$$

We may assume that in each series — the one with dominant paternal and with dominant maternal type — the variability will be independent of the deviation of the father (respectively the mother) from the average type. We will designate with s the variability of children with dominant paternal (or maternal) type who are descendants of fathers and mothers who have all the same deviations x and y respectively from the averages of fathers and mothers. When the average of the squares of the deviations of such a series of children is taken from the grand average, not from their proper average $[z]$, the average of these squares will be for the series with dominant paternal trait

$$s^2 + [z]^2 = s^2 + \left(r_{cf}\frac{\sigma_c}{\sigma_f}x + r_{cm}\frac{\sigma_c}{\sigma_m}y\right)^2$$

when we take the average of these values for all the values of x and y we find

$$[s^2 + [z]^2] = s^2 + (r^2_{cf} + r^2_{cm})\,\sigma^2_c \quad n_1 \text{ cases.}$$

In the same way we find for the series of children with dominant maternal trait

$$[s^2 + [z]^2] = s^2 + (r'^2_{cf} + r'^2_{cm})\,\sigma^2_c \quad n_2 \text{ cases.}$$

Here it is assumed that the variability of the series with dominant paternal and with dominant maternal type is the same. Since both groups constitute the whole series, we have

(4) $$\sigma_c^2 = s^2 + \frac{\sigma_c^2}{n}\{n_1(r_{cf}^2 + r_{cm}^2) + n_2(r_{cf}'^2 + r_{cm}'^2)\}$$

We will make two assumptions:

$$\text{I.}\quad n_1 = n_2 = \tfrac{1}{2}n.$$
$$r_{cf} = r_{cf}'$$
$$r_{cm} = r_{cm}'$$
$$\text{II.}\quad n_1 = n_2 = \tfrac{1}{2}n$$
$$r_{cf}' = r_{cm}' = 0.$$

For assumption I the series with dominant paternal and maternal traits assume the same form, *i. e.*, neither trait is dominant, and we have ordinary correlation.

(1*) $$[z] = r_{cf}\frac{\sigma_c}{\sigma_f}x + r_{cm}\frac{\sigma_e}{\sigma_m}y$$

(2*) $$\frac{[xz]}{\sigma_c\sigma_f} = r_{cf}$$

(3*) $$\frac{[yz]}{\sigma_c\sigma_m} = r_{cm}$$

(4*) $$s^2 = \sigma_c^2(1 - r_{cf}^2 - r_{cm}^2).$$

For assumption II we find

(1**) $$[z] = \tfrac{1}{2}r_{cf}\frac{\sigma_c}{\sigma_f}x + \tfrac{1}{2}r_{cm}\frac{\sigma_c}{\sigma_m}y$$

(2**) $$2\frac{[xz]}{\sigma_c\sigma_f} = r_{cf}$$

(3**) $$2\frac{[yz]}{\sigma_c\sigma_m} = r_{cm}$$

(4**) $$s^2 = \sigma_c^2(1 - \tfrac{1}{2}r_{cf}^2 - \tfrac{1}{2}r_{cm}^2).$$

We will now proceed to determine the average value for children of couples with the deviations x and y, and the variability of these children measured from the average.

According to assumption I

$$[z] = \frac{[xz]}{\sigma_f^2} x + \frac{[yz]}{\sigma_m^2} y \tag{5*}$$

Since there is no difference between the series with dominant paternal and with dominant maternal traits, the variations of the children will be distributed regularly around their average type. Therefore,

$$s^2 = \sigma_c^2 \left(1 - \frac{[xz]^2}{\sigma_c^2 \sigma_f^2} - \frac{[yz]^2}{\sigma_c^2 \sigma_m^2} \right) \tag{6*}$$

According to assumption II

$$[z] = \frac{[xz]}{\sigma_f^2} x + \frac{[yz]}{\sigma_m^2} y \tag{5**}$$

The series with dominant paternal type will vary around the value

$$[z] = 2 \frac{[xz]}{\sigma_f^2} x$$

Its variability, measured from this average, will be

$$s^2 = \sigma_c^2 \left(1 - 2 \frac{[xz]^2}{\sigma_c^2 \sigma_f^2} - 2 \frac{[yz]^2}{\sigma_c^2 \sigma_m^2} \right)$$

If the variability is determined from the average of both series, the one with dominant paternal and the one with dominant maternal traits, its value will be

$$s_{xy}^2 = \sigma_c^2 \left(1 - 2 \frac{[xz]^2}{\sigma_c^2 \sigma_f^2} - 2 \frac{[yz]^2}{\sigma_c^2 \sigma_m^2} \right) + \left(\frac{[yz]}{\sigma_m^2} y - \frac{[xz]}{\sigma_f^2} x^2 \right) \tag{6**}$$

Since the variability of the series with dominant maternal traits, gives the same value, the formula presents the variability of children for parents whose deviations from the average are respectively x and y. We find thus, that in case of assumption II the variability of children increases with increasing difference between the parents.

The available material is so small that it is not possible to determine the formula that fits the observations most accurately. It is, however, easy to show that the variability increases with increasing difference between the parents. It appears that in both

series the expected average of the children of parents with the deviation x and y is the same. The following coefficients of correlation were calculated from the series of observations:

$$\frac{[xz]}{\sigma_c \sigma_f} = 0.30$$

$$\frac{[yz]}{\sigma_c \sigma_m} = 0.36$$

or approximately 0.33 for either. σ_c, σ_m, σ_f were found respectively 3.0, 3.2, 3.3 or approximately equal.

By substituting

$$\frac{[xz]}{\sigma_c \sigma_f} = \frac{[yz]}{\sigma_c \sigma_m} = r$$

and

$$\sigma_c = \sigma_m = \sigma_f = \sigma$$

we have according to I

(7*) $$s^2 = \sigma^2(1 - 2r^2).$$

According to II

(7**) $$s^2_{xy} = \sigma^2 (1 - 4r^2) + r^2(y - x)^2,$$

and for both

(8) $$[z] = r(x + y).$$

The calculation of the variability of the children from the point $[z]$ found in (8) is, of course, very inaccurate on account of the small number of children in each family; but since the same inaccuracies prevail for all differences between parents, it seems justifiable to compare the results for different values of x and y.

The variabilities thus obtained give decisive results. I have calculated the correlation between the standard variation of the children of each couple and the difference between the parents. It may be well to give the table of these differences and variabilities in full.

Absolute difference between deviations of parents $(y-x)$.	Square of standard variation of children (s^2_{xy}).	Absolute difference between deviations of parents $(y-x)$.	Square of standard variation of children (s^2_{xy}).
11.3	25.9	2.7	13.0
10.2	21.9	2.7	8.3
10.1	13.0	2.5	5.3
9.9	17.5	2.5	3.1
9.8	22.0	2.4	13.8
6.9	18.8	2.3	14.4
6.5	6.3	2.2	3.3
6.0	3.1	2.0	2.1
5.2	23.5	1.9	3.2
5.0	5.2	1.9	1.3
4.4	2.5	1.3	16.1
4.3	11.7	1.3	4.3
4.0	4.4	1.0	2.6
3.9	8.5	1.0	4.6
3.7	3.3	1.0	4.4
3.7	9.3	0.9	9.7
3.6	15.9	0.9	0.9
3.1	0.5	0.9	13.5
3.1	0.3	0.9	2.7
3.0	7.0	0.8	1.0
3.0	13.1	0.6	3.0
2.9	0.7	0.4	1.8
2.9	6.3	0.2	5.3
2.8	3.0	0.2	1.5

An inspection of this table shows clearly that the variability of children increases with increasing difference of parents. The index of correlation between the two is + 0.67, which corresponds to an index of regression of + 1.6. That is to say, the deviation from the average of the square standard deviation of children equals 1.6 times the deviation from the average difference of parents. Thus it will be seen that the variability of children is not by any means constant, and therefore our first assumption (I) is not tenable.

A calculation of the values $s^2_{xy} - s^2(y - x)^2$ from the table of observations, and a correlation of these values with the differences between parents give an index of correlation of + 0.14 and an index of regression as above of + 0.2. These amounts are so small that we may well assume that in a more extended series they would

still more approach the theoretical value zero. The agreement with our second assumption (II) seems satisfactory.

We conclude from these data that heredity of the cephalic index in individuals of the same race does not depend on the midparental value of the index, but that one-half of the children resemble in regard to this trait the father, and the other half the mother. According to equations (2**) and (3**) the index of correlation between father and child in the series with dominant paternal type, and between mother and child in the series with dominant maternal type is 0.66, so that we may say that *one-half of the children of a couple belonging to a certain race have a type the average of which is equal to the average of twice the father's type and once the racial type, while the other half have an average equal to twice the mother's type and once the racial type.*

It is likely that this law is also an approximation only, but it agrees with the facts much better than the assumption of the reproduction of the midparental type, in which case the offspring would all form one series the average of which would be equal to the average of the father's, the mother's, and the racial type.

It seems plausible that similar laws prevail in regard to other measurements, but probably in such a manner that the offspring resembles in one respect his father, in another his mother.

The data here given do not show what the law of heredity of the cephalic index may be when father and mother belong to different races. It must also be remembered that other measurements may follow different laws. A case of this kind is the stature of Americo-European half-bloods, which, as I have previously shown,[1] exceeds that of both parental types.

[1] *Popular Science Monthly*, October, 1894.